Question 11 (8 marks)

(a) Determine the equations of all asymptotes of the graph of when

(i) . (2 marks)

(ii) . (2 marks)

<EFOFEX>
id:fxd{ceb15cb5-42b9-4826-b51c-b80f23c89254}

FXData:

</EFOFEX>

(b) The graph of is shown  
in the diagram, together with its  
three asymptotes.  
  
The defining rule is given by  
  
 and are positive  
integer constants.  
  
Determine, with brief reasons, the value of and . (4 marks)

Question 11 (8 marks)

(a) Determine the equations of all asymptotes of the graph of when

(i) . (2 marks)

|  |
| --- |
| Solution |
| Asymptotes: . |
| Specific behaviours |
| ✓ horizontal asymptote  ü all asymptotes |

(ii) . (2 marks)

|  |
| --- |
| Solution |
| Asymptotes: , . |
| Specific behaviours |
| ✓ oblique asymptote  ü all asymptotes |

<EFOFEX>
id:fxd{ceb15cb5-42b9-4826-b51c-b80f23c89254}

FXData:

</EFOFEX>

(b) The graph of is shown  
in the diagram, together with its  
three asymptotes.  
  
The defining rule is given by  
  
 and are positive  
integer constants.  
  
Determine, with brief reasons, the value of and . (4 marks)

|  |
| --- |
| Solution |
| Asymptote .  Root at .  Asymptote .  Asymptote . |
| Specific behaviours |
| ✓üüü each value with appropriate reason |

Question 14 (9 marks)

The graph of is shown on the left-hand axes in the diagram below.

<EFOFEX>
id:fxd{9a1561dd-a499-4f7f-a82a-4331e55ca192}

FXData:

</EFOFEX>

(a) Sketch the graph of on the right-hand axes in the diagram. (5 marks)

(b) Solve the following equations.

(i) . (1 mark)

(ii) . (1 mark)

(iii) . (2 marks)

Question 14 (9 marks)

The graph of is shown on the left-hand axes in the diagram below.

<EFOFEX>
id:fxd{720dddee-b84e-49fe-8884-cfbcbbdb8e64}

FXData:

</EFOFEX>

|  |
| --- |
| Solution (a) |
| See graph. |
| Specific behaviours |
| ✓ indicates all vertical asymptotes  ü correct graph for  ü correct graph for  ü correct graph for  ü correct graph for |

(a) Sketch the graph of on the right-hand axes in the diagram. (5 marks)

(b) Solve the following equations.

(i) . (1 mark)

|  |
| --- |
| Solution |
| For . |
| Specific behaviours |
| ✓ correct solution set |

(ii) . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct solution set |

(iii) . (2 marks)

|  |
| --- |
| Solution |
| Roots and intervals where : |
| Specific behaviours |
| ✓ includes roots  ü correct solution set |

Question 10 (6 marks)

The graph of is shown.

<EFOFEX>
id:fxd{09a1f42a-e827-4e37-8b84-bf2065a46ddd}

FXData:

</EFOFEX>

Using the set of axes provided, draw the graph of

(a) . (2 marks)

<EFOFEX>
id:fxd{8b35a749-548f-42c7-a328-c4d6d2ba8670}

FXData:

</EFOFEX>

(b) . (2 marks)

<EFOFEX>
id:fxd{8b35a749-548f-42c7-a328-c4d6d2ba8670}

FXData:

</EFOFEX>

(c) . (2 marks)

<EFOFEX>
id:fxd{8b35a749-548f-42c7-a328-c4d6d2ba8670}

FXData:

</EFOFEX>

Question 10 (6 marks)

The graph of is shown.

<EFOFEX>
id:fxd{09a1f42a-e827-4e37-8b84-bf2065a46ddd}

FXData:

</EFOFEX>

Using the set of axes provided, draw the graph of

(a) . (2 marks)

<EFOFEX>
id:fxd{2c467a90-cd0f-40fb-87b5-dd1ea147f54a}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See graph: reflect lines below -axis upwards |
| Specific behaviours |
| ✓ section to left of -axis  ü section to right of -axis |

(b) . (2 marks)

<EFOFEX>
id:fxd{b9ed972b-1736-468c-a371-04dded84f757}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See graph: reflect answer to (a) in -axis |
| Specific behaviours |
| ✓ section to left of -axis  ü section to right of -axis |

(c) . (2 marks)

<EFOFEX>
id:fxd{ebda2d8c-321b-4efb-a9b6-e165574daa4c}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See graph: reflect lines left of -axis to right |
| Specific behaviours |
| ✓ lines as shown  ü features when |

Question 17 (6 marks)

Consider the function , where and are positive constants.

The graph of cuts the -axis at , has a horizontal asymptote with equation and has as one of its vertical asymptotes.

(a) Determine . (3 marks)

(b) Now consider the graph of . State the

(i) equation of its horizontal asymptote. (1 mark)

(ii) -axis intercepts. (1 mark)

(iii) equations of its vertical asymptotes. (1 mark)

Question 17 (6 marks)

Consider the function , where and are positive constants.

The graph of cuts the -axis at , has a horizontal asymptote with equation and has a vertical asymptote with equation .

(a) Determine . (3 marks)

|  |
| --- |
| Solution |
| Horizontal asymptote .  Vertical asymptote |
| Specific behaviours |
| ✓ obtains value of one constant  ü obtains value of second constant  ü correct value of |

(b) Now consider the graph of . State the

(i) equation of its horizontal asymptote. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct equation |

(ii) -axis intercepts. (1 mark)

|  |
| --- |
| Solution |
| Vertical asymptotes roots: . |
| Specific behaviours |
| ✓ correct intercepts |

(iii) equations of its vertical asymptotes. (1 mark)

|  |
| --- |
| Solution |
| Roots vertical asymptotes: and . |
| Specific behaviours |
| ✓ correct equations |

Question 11 (9 marks)

Let and .

(a) State the domain and range of . (3 marks)

(b) Determine and state its range. (3 marks)

(c) Determine an expression for and state the domain for which the composite function is defined. (3 marks)

Question 11 (9 marks)

Let and .

(a) State the domain and range of . (3 marks)

|  |
| --- |
| Solution |
| For natural domain we require .  Hence and .  Range will be all values of where . Hence |
| Specific behaviours |
| ✓ indicates at least one required condition / inequality  ü correct domain  ü correct range |

(b) Determine and state its range. (3 marks)

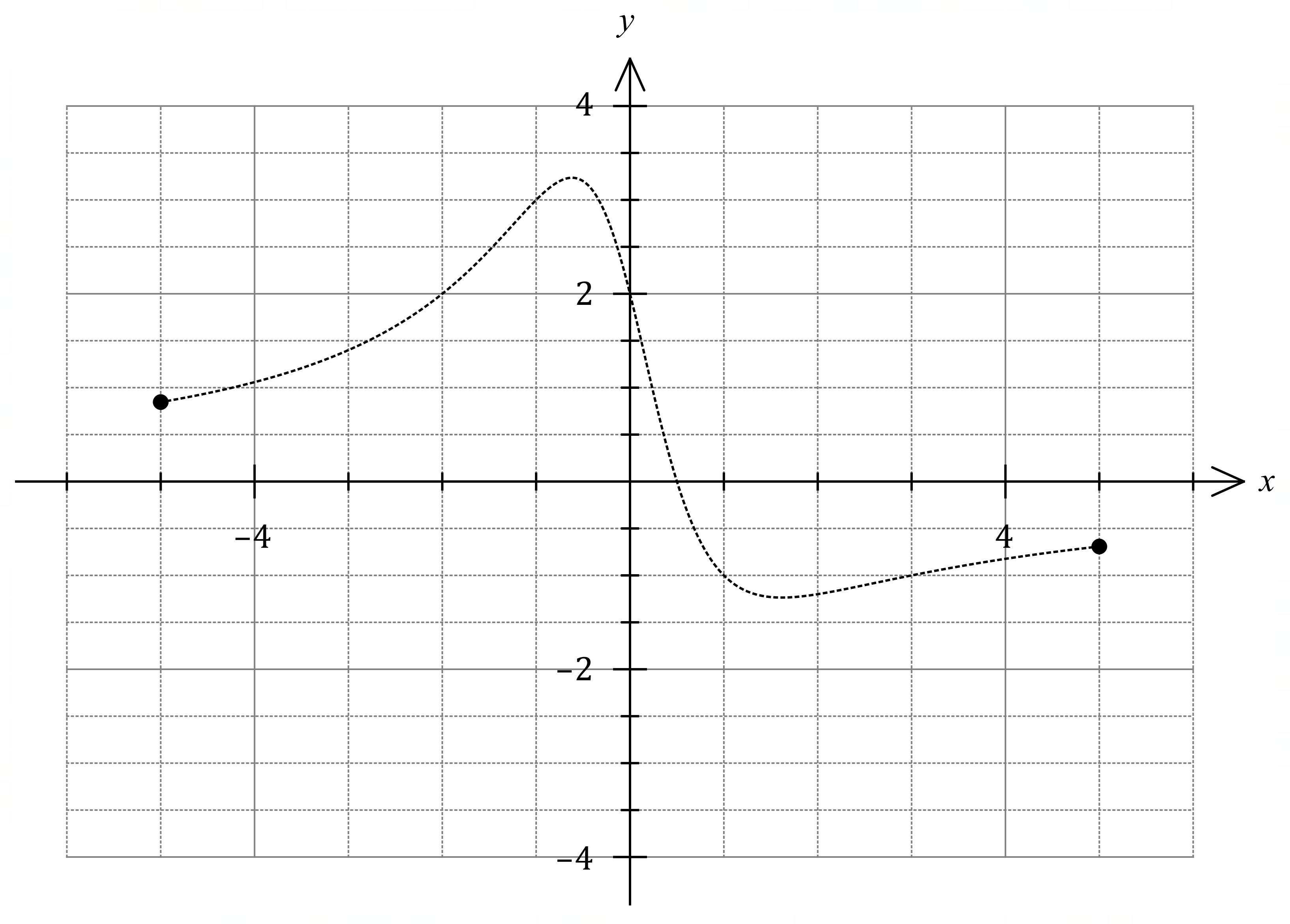
|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates appropriate steps to obtain inverse  ü correct inverse (factored or expanded form)  ü correct range |

(c) Determine an expression for and state the domain for which the composite function is defined. (3 marks)

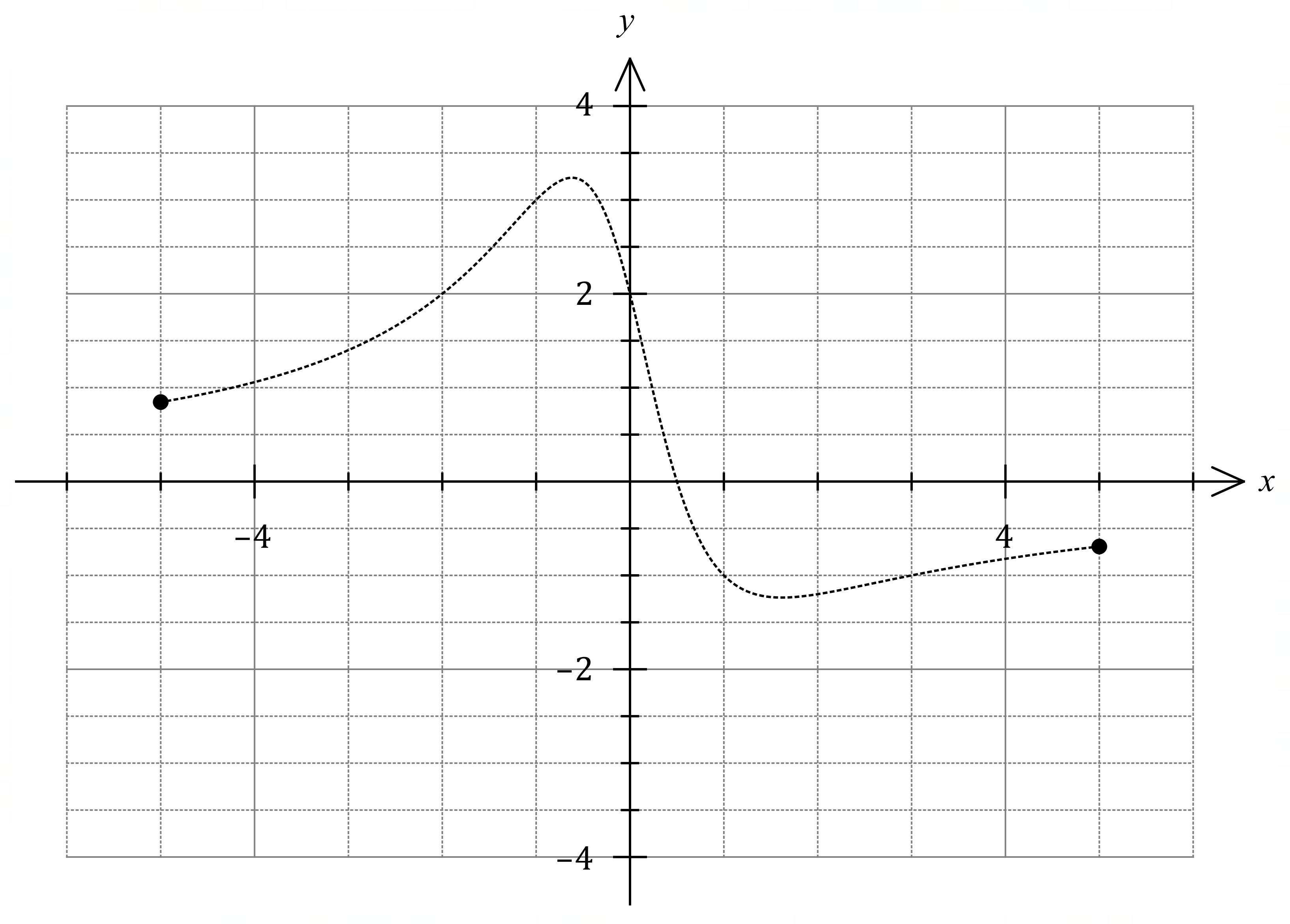
|  |
| --- |
| Solution |
| For the domain we require |
| Specific behaviours |
| ✓ correct expression for composite function  ü indicates required condition  ü correct domain |

Question 12 (6 marks)

(a) The dotted curve on the axes below is the graph of . On the same axes, sketch the graph of . (4 marks)

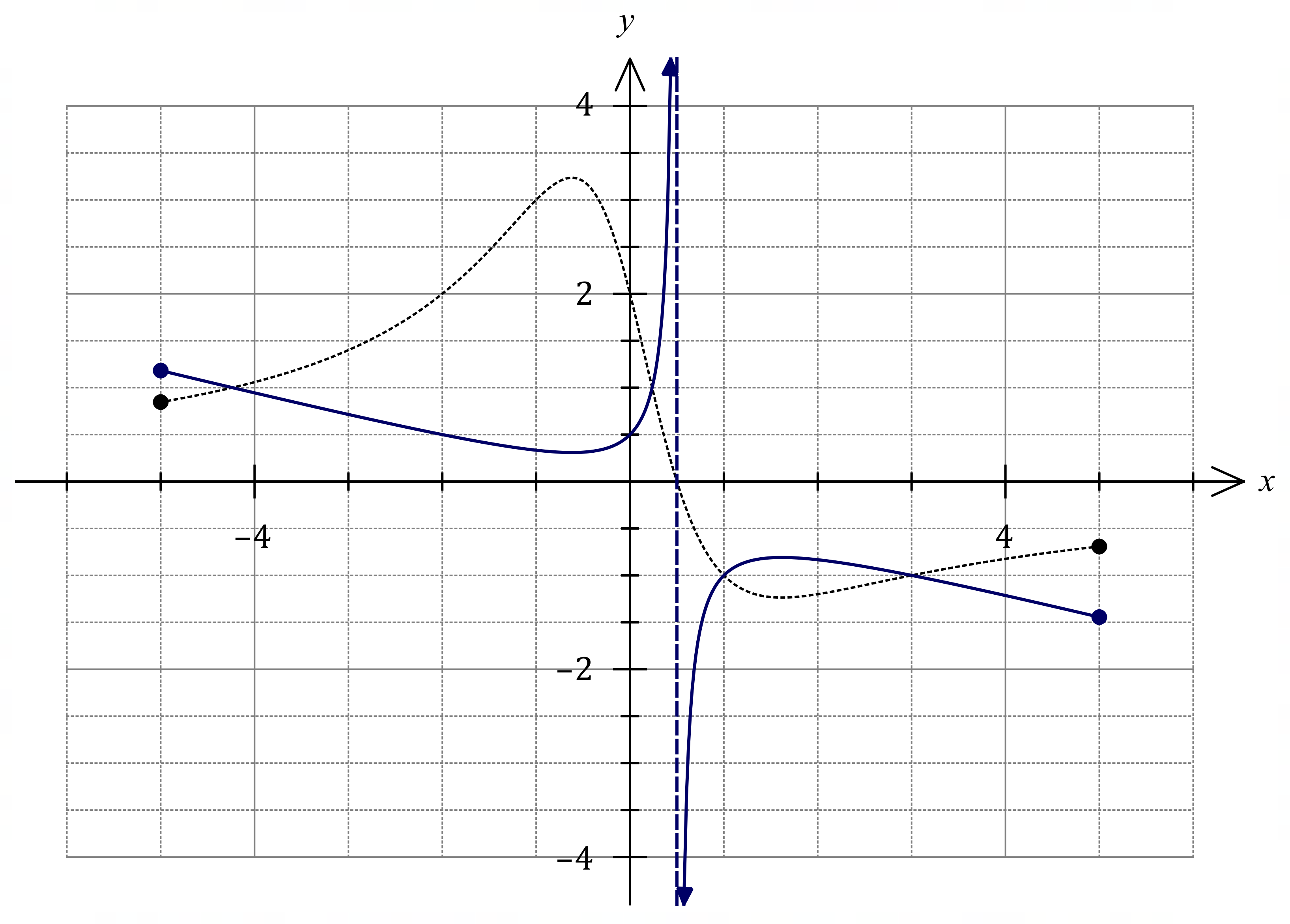


(b) The dotted curve on the axes below is the graph of . On the same axes, sketch the graph of . (2 marks)



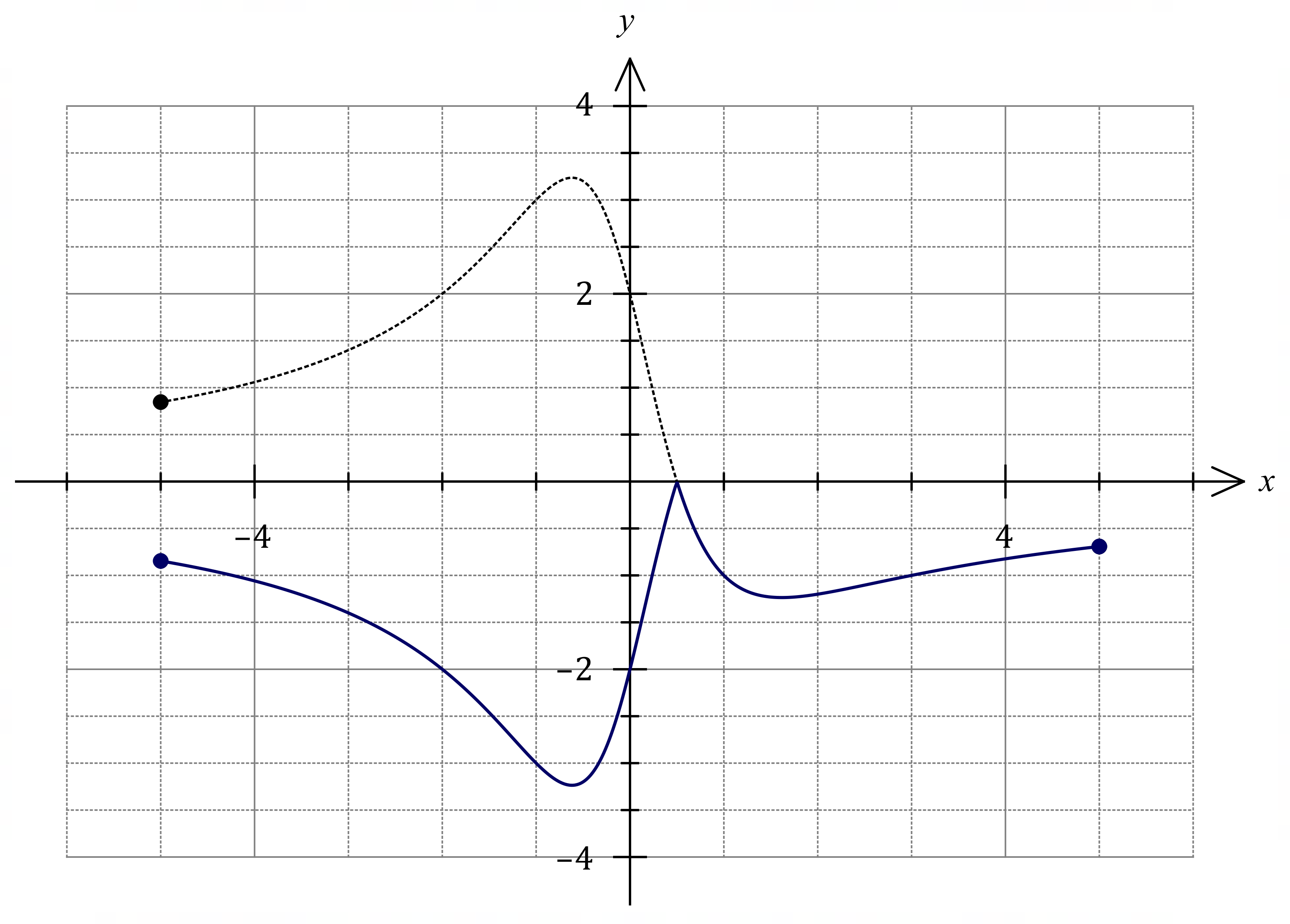
Question 12 (6 marks)

(a) The dotted curve on the axes below is the graph of . On the same axes, sketch the graph of . (4 marks)



|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ asymptote and curvature nearby  ü and curve meet on lines  ü maximum and curve to endpoint  ü minimum, intercept, curve to endpoint |

(b) The dotted curve on the axes below is the graph of . On the same axes, sketch the graph of . (2 marks)



|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ cusp and curve to right  ü reflects curve to left of root |

Question 18 (6 marks)

(a) State the equations of all asymptotes of the graph of . (2 marks)

(b) Let .

The graph of has no roots, a -intercept of and two asymptotes (with equations and ). Determine the value of each of the constants and .

(4 marks)

Question 18 (6 marks)

(a) State the equations of all asymptotes of the graph of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ horizontal asymptote   both vertical asymptotes |

(b) Let .

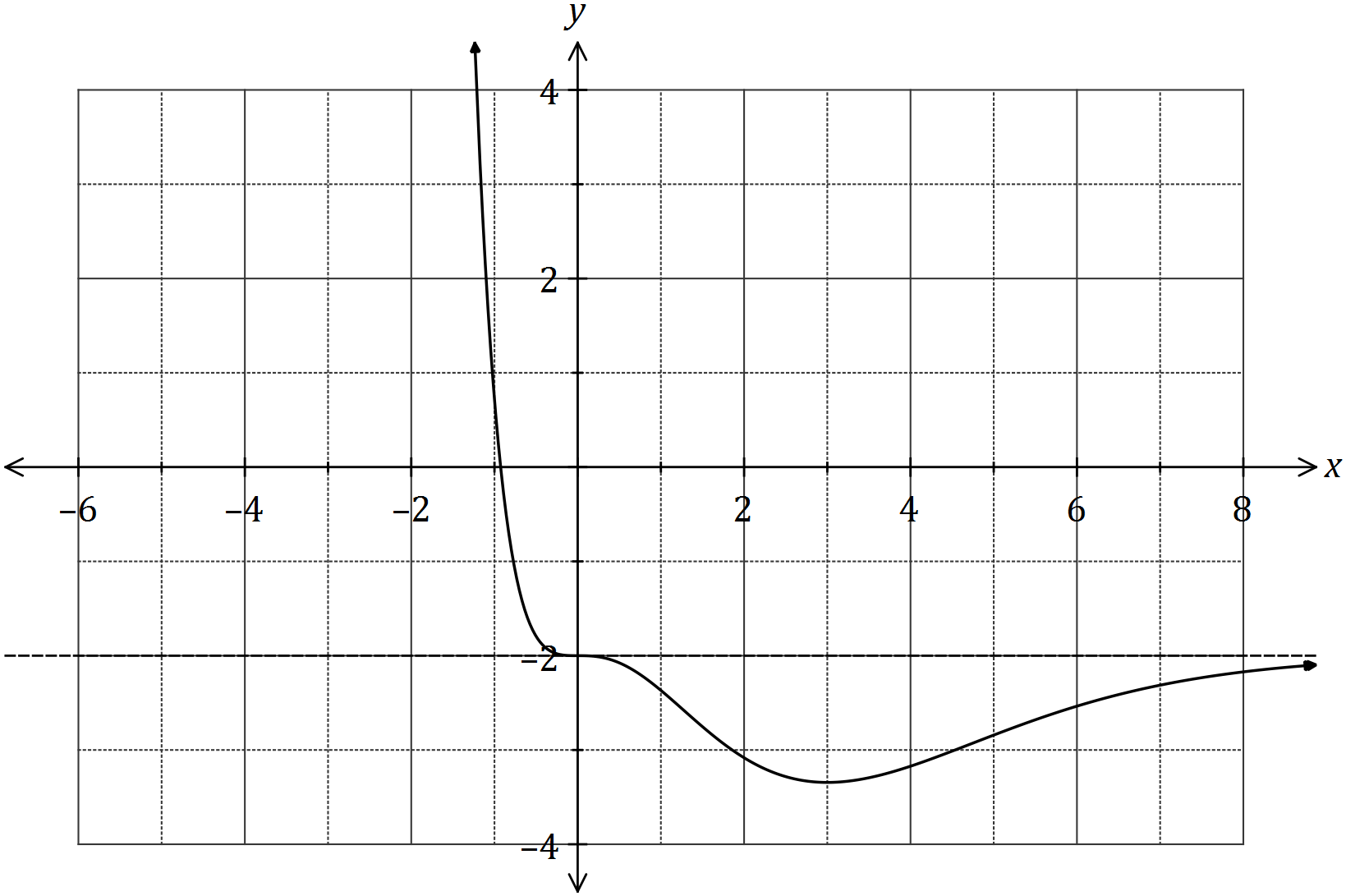
The graph of has no roots, a -intercept of and two asymptotes (with equations and ). Determine the value of each of the constants and .

(4 marks)

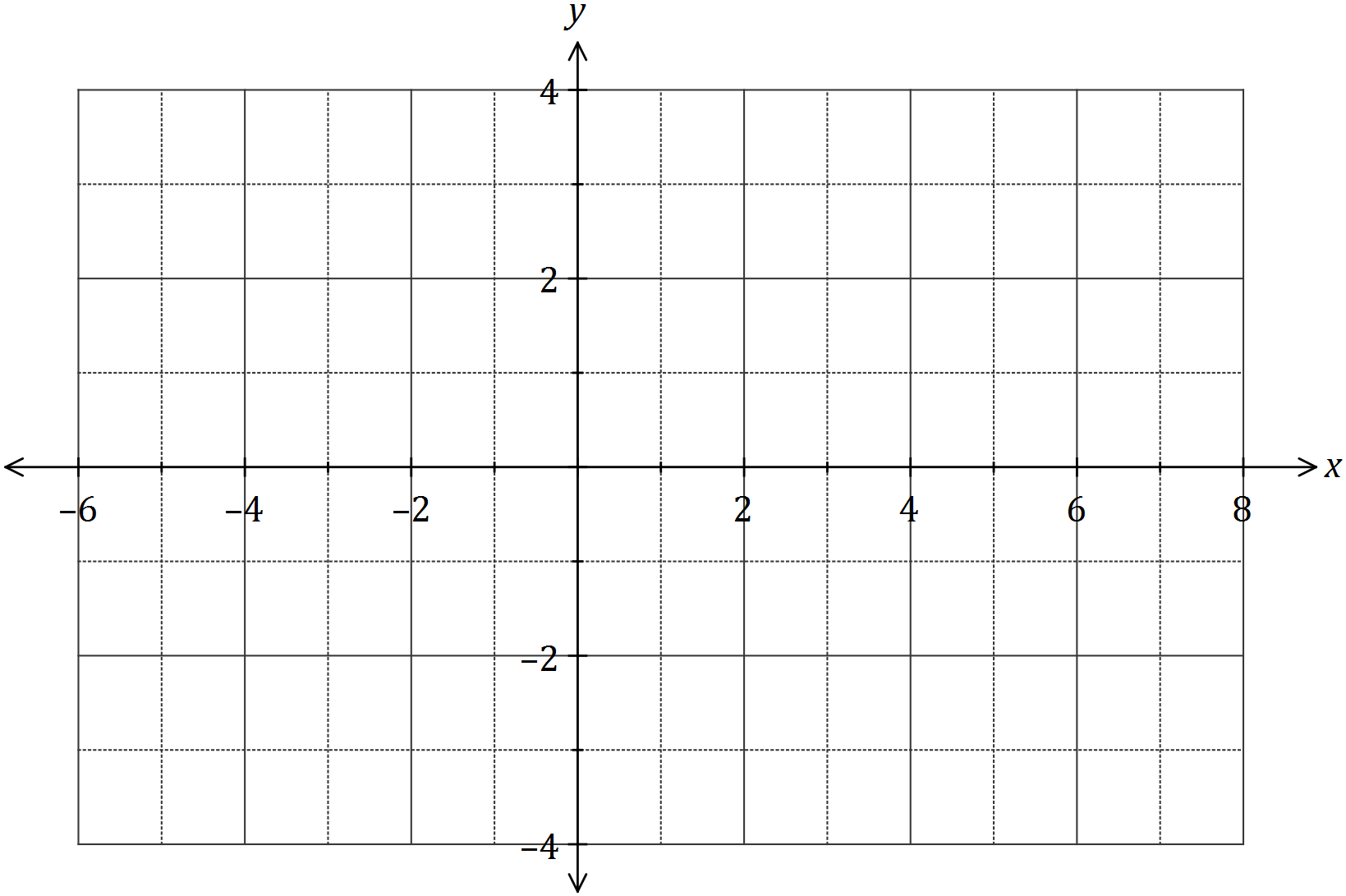
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses asymptotes to form equation   uses intercept to determine   combines into single fraction   correctly states all four values  *(Max  if incorrect)* |

Question 14 (8 marks)

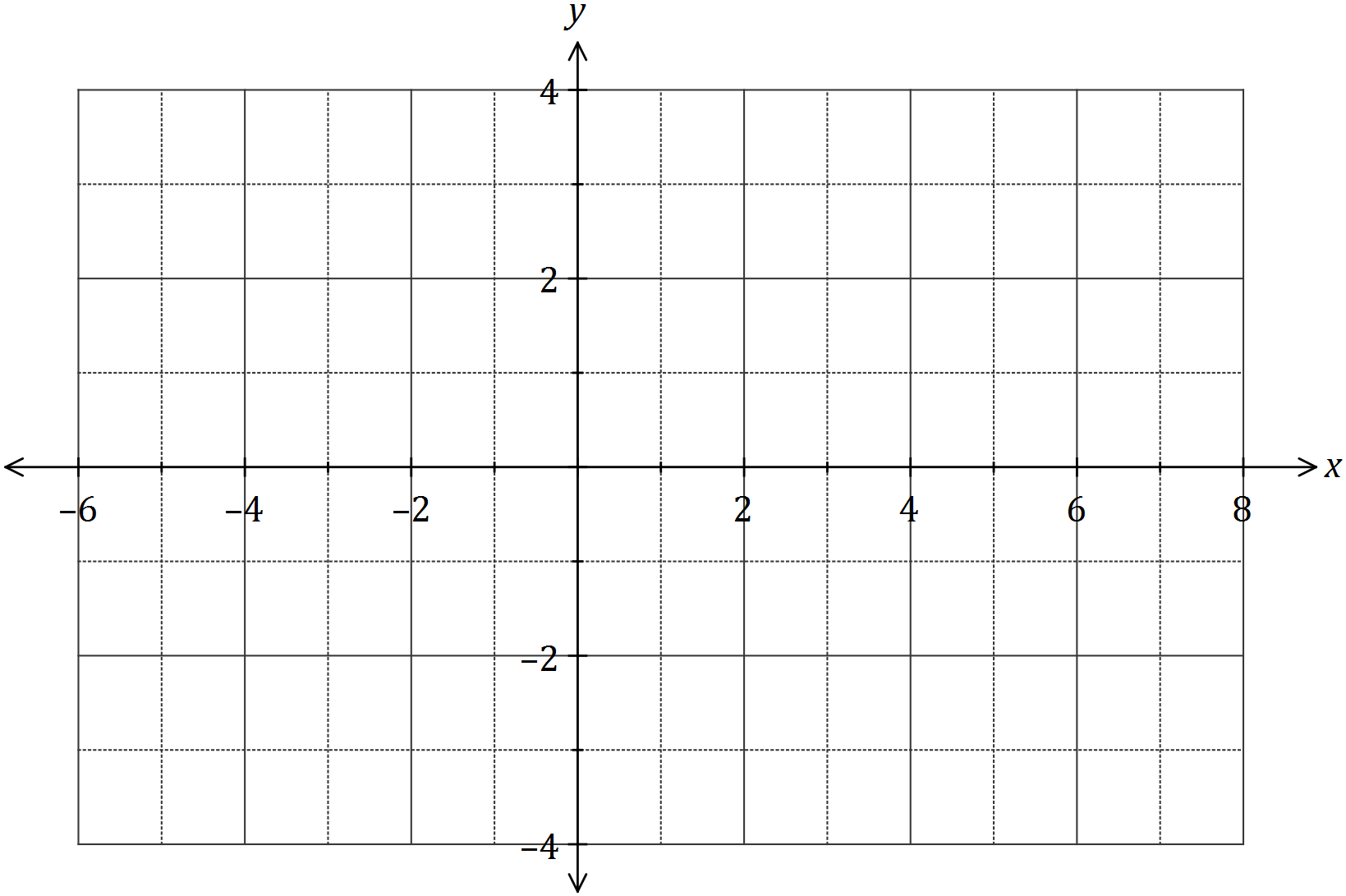
The graph of has asymptote with equation , and is shown below.



(a) Sketch the graph of on the axes below. (5 marks)

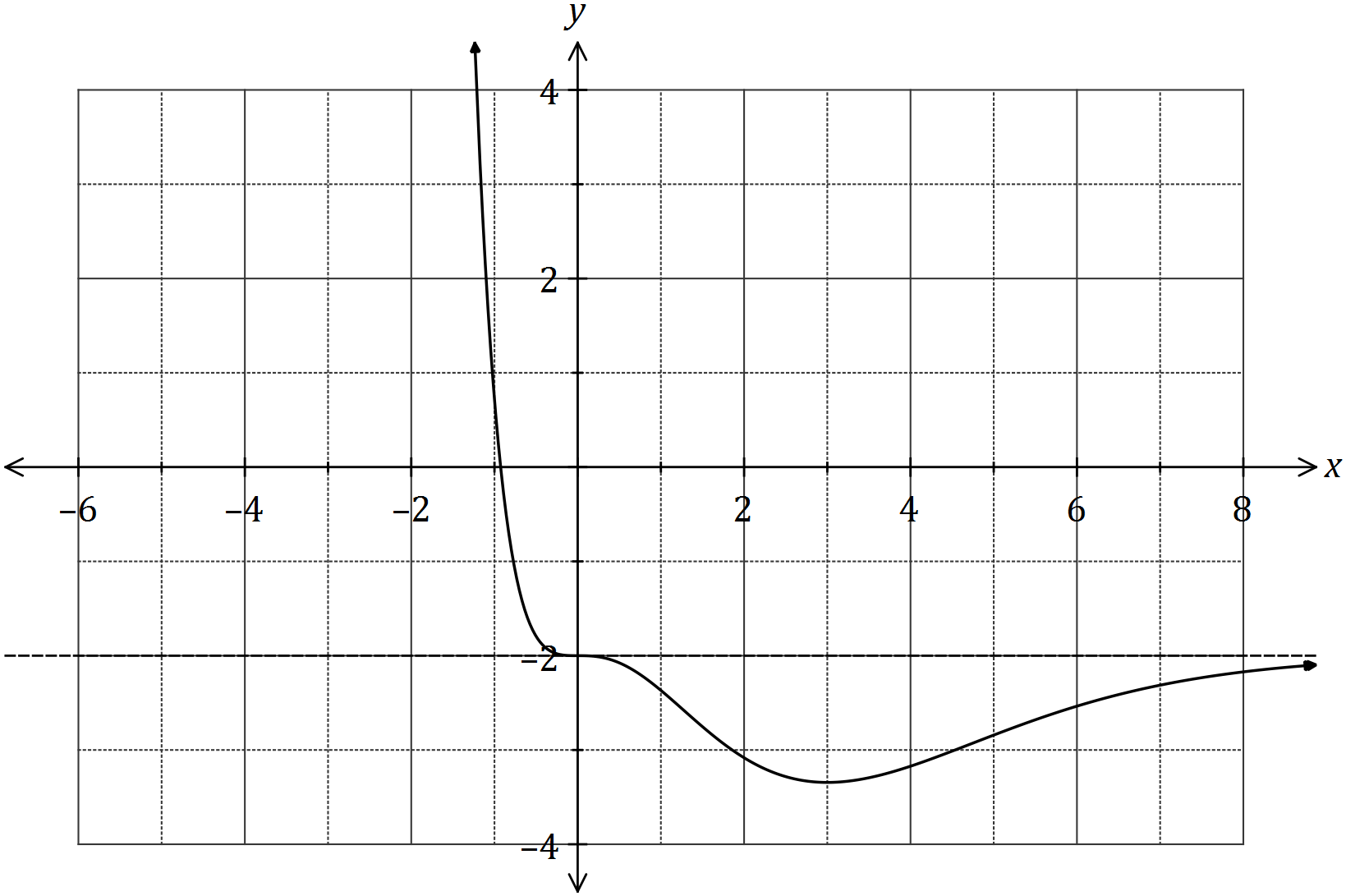


(b) Sketch the graph of on the axes below. (3 marks)

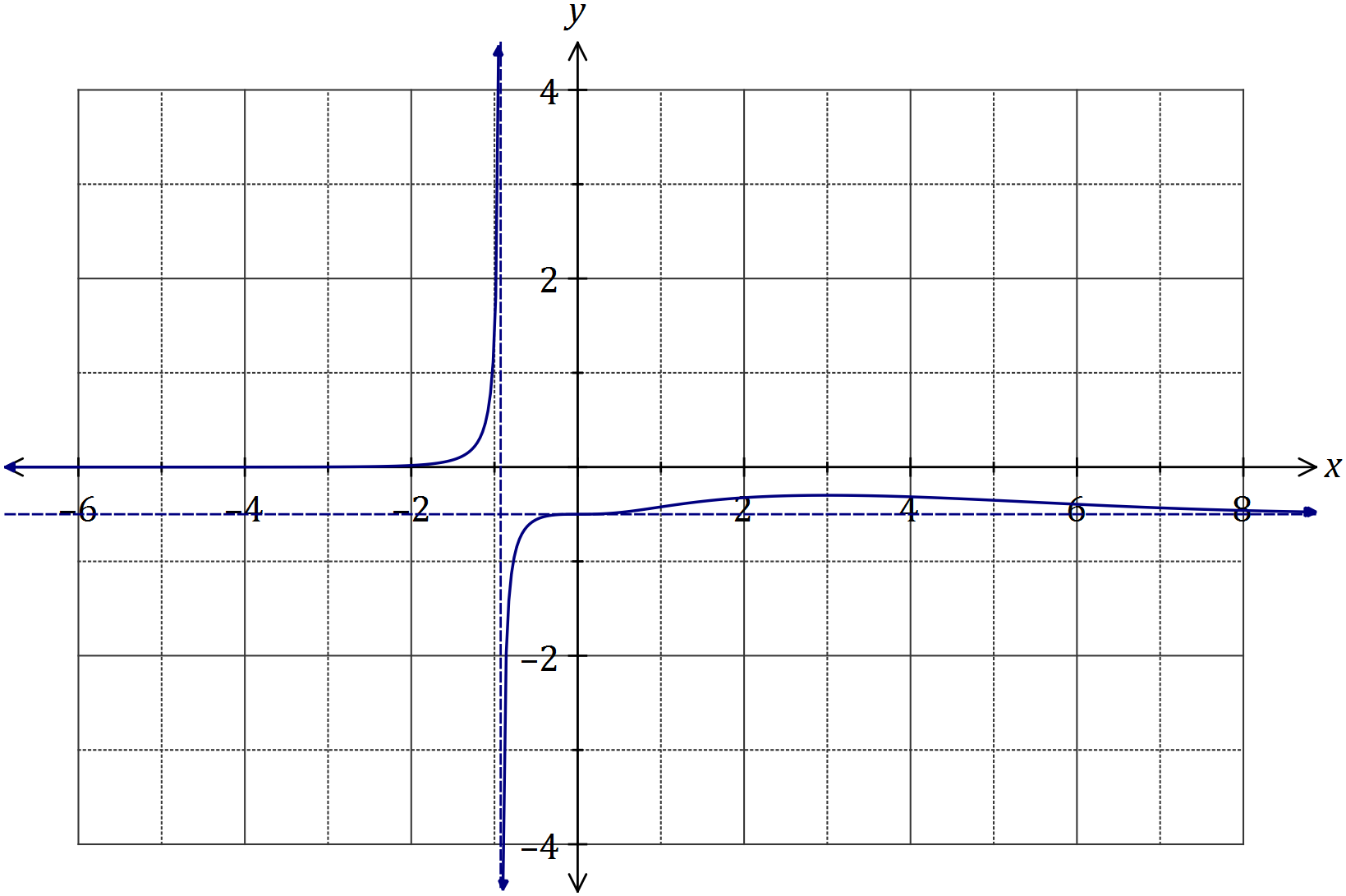


Question 14 (8 marks)

The graph of has asymptote with equation , and is shown below.

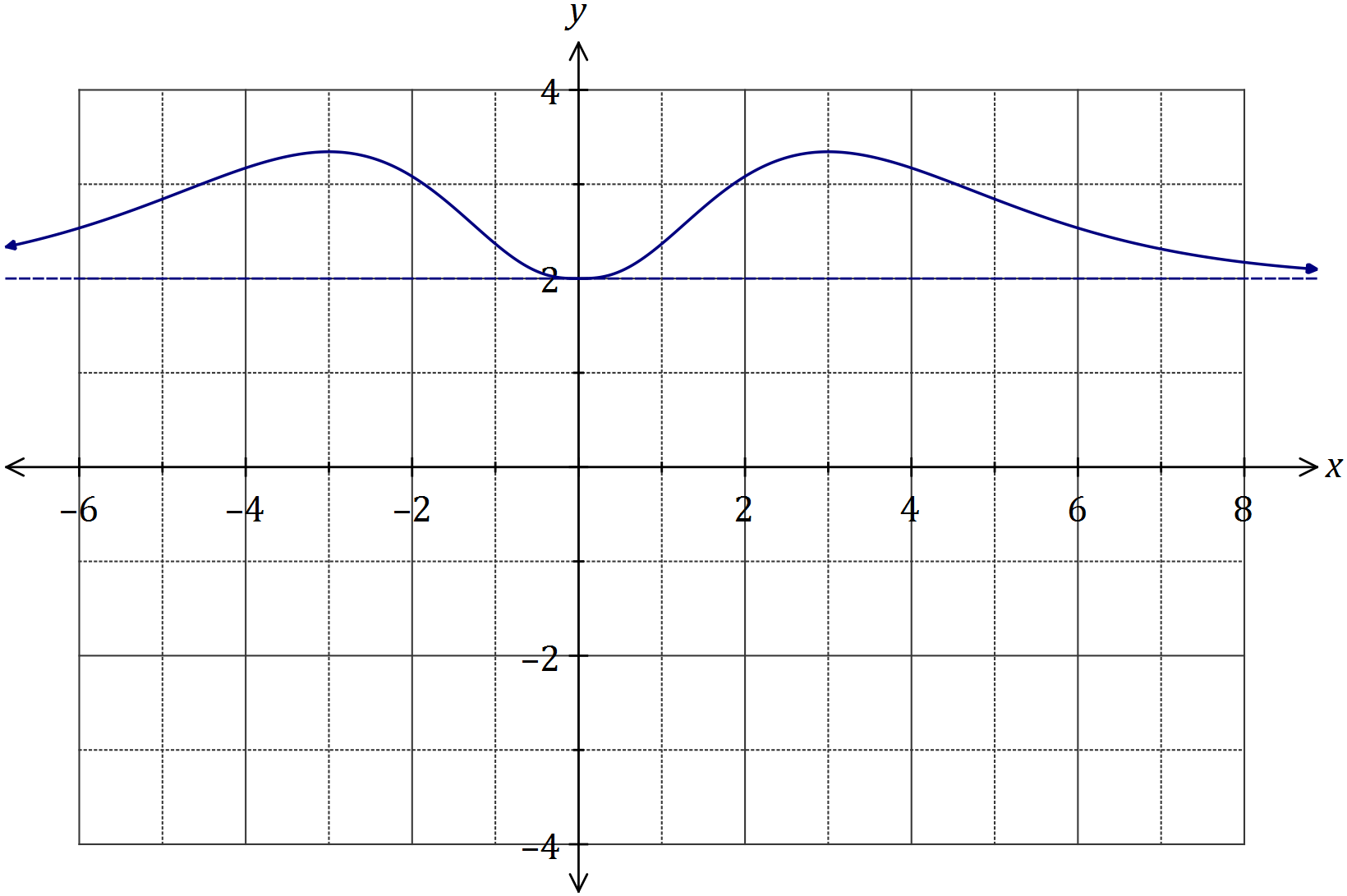


(a) Sketch the graph of on the axes below. (5 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ indicates vertical asymptote  ✓ indicates correct curvature and behaviour for  ✓ indicates as , approaches horizontal asymptote  ✓ indicates point of inflection at  ✓ indicates local maximum at about |

(b) Sketch the graph of on the axes below. (3 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ indicates maxima at  ✓ indicates symmetry about  ✓ indicates -intercept at |

Question 16 (8 marks)

Consider the function .

(a) Using your calculator, or otherwise, write down the exact area bounded by and the lines and . (2 marks)

(b) can be written in the form . State the functions and . (2 marks)

(c) Show how to use integration to obtain the answer to (a) without a CAS calculator.

(4 marks)

Question 16 (8 marks)

Consider the function .

(a) Using your calculator, or otherwise, write down the exact area bounded by and the lines and . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes integral  ✓ evaluates in exact form |

(b) can be written in the form . State suitable functions for and .

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states  ✓ states |

(c) Show how to use integration to obtain the answer to (a) without a CAS calculator.

(4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ shows and evaluates integral under  ✓ uses substitution for integral of  ✓ evaluates integral of  ✓ subtracts integrals |